H∞/μ - SYNTHESIS for CUK CONVERTOR CIRCUIT CONTROLLER

Dr. Ali H. Ahmed
Assis. Professor
Electr. Eng.
College of Eng.
Univ. of Mosul

Dr. Majid K. Alkhatat
Lecturer
Dept. of Electrical. & Electronics Eng.
Univ. of Technology

Eng. Ahmed A. J. Abdullah
Dept. of Electrical.& Electronics Eng.
Univ. of Technology

Abstract

The present work deals with robust stability analysis and synthesis of Cuk converter circuit .Mathematical model in state space form is developed for the circuit .A general system matrix is derived for the system with perturbations in a chosen elements values in the system. H_∞ - design technique is applied to design a feedback controller for the system with its nominal values. A μ - synthesis technique is used to design a controller for the system to guaranty its robust stability .The system with designed controller of the two above techniques, is investigated. It is observed that the controller obtained from μ - synthesis technique guaranties the robustness of the Cuk converter circuit with *25%* simultaneous uncertainties in all perturbed elements.

Key wards: Robust Stability, H_∞ - Design Technique, μ - Synthesis Technique

CUK [illegible] H∞/μ

[illegible]

[illegible] (Cuk) [illegible] (H_∞) [illegible] (μ) [illegible] (μ , H_∞) [illegible] (Cuk) [illegible] (μ) [illegible] %25 [illegible]

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1 – Introduction

Design of controllers for uncertain plants with guarantied closed - loop stability and performance has been the focus of vigorous researchers for about two decades .Most of the researches in robust control were paying attention on H_∞ like problems .Many practical problems do not readily fit the standard H_∞ problem setup ,since the involved model uncertainty is structured rather than unstructured ,which causes any H_∞ controller design to be potentially conservative and thus limits the obtainable performance of the closed -loop systems.[1,2]. For example J.C.Doyle & G. Stein [3],showed in their paper that even though the control law and the state estimation law are optimal, there is no guarantee that the combined linear quadratic Gaussian (LQG) scheme possesses equivalent fine properties. J.C.Doyle & K.Glover [4] used state space solution for H_∞ control problems, based on the minimization of the H_∞- norm of the closed - loop transfer function between disturbance inputs and performance outputs .The H_∞ technique is capable of obtaining robust multivariable design for a wide spectrum of single block uncertainties and systems. While H_∞ theory constitute a considerable innovation in robust control design .It still does not take into account the possible structure in the uncertainty .Thus for multiple uncertainties at different locations in the plant , an H_∞ design is too conservative, which can lead to controllers unable to satisfy performance specifications

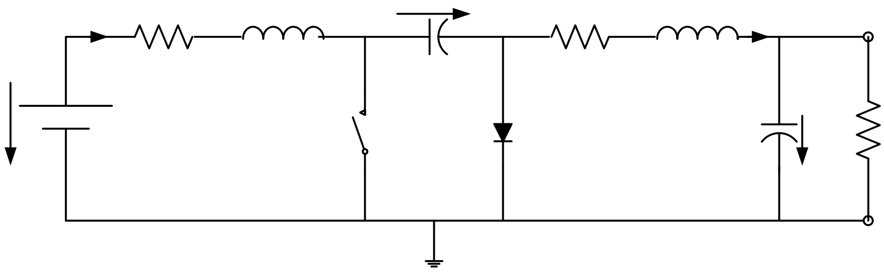
DC-DC converters received deep attentions in the field of robust performance and robust stability studies. D.Cortes and J. Alvarez [5] , proposed a composite controller for the DC-DC Buck converter. The controller developed is intended to be used in high performance applications and yet keeping a simple configuration. The paper deals with the performance only and not working on the parameter uncertainty. R. Tymerski [6] , presented a procedure for the design of robust controller for Boost converter circuit . Robust stability and performance is guaranteed with respect to uncertainties tolerances in the power components, a satisfactory results have been got. S. Buso[7] , proposed the structured singular value (μ) approach to the problem of designing an output voltage regulator for a Buck - Boost converter. μ- controller is compared with the conventional PI regulator by testing robust stability and performance of both. A reduced order controller was derived, whose practical implementation is feasible. The paper did not consider the effect of output disturbances. Y. Fuad . . .etc.[8], showed a linear controller for a multi- frequency model of a pulse - width - modulated Cuk - converter .The paper did not deal with perturbation in parameters and did not use the μ - synthesis technique

In this work, a hybrid scheme between H_∞ loop - shaping and a μ synthesis is used to design a robust controller for DC-DC Cuk converter circuit.

2 - Cuk Converter Circuit

The Cuk converter circuit is shown in Fig(1) .This circuit performs a dc conversion function. It can either increase or decrease the magnitude of the output voltage with respect to the input and it inverts its polarity [9].

Its principle of operation is as follows: During t_{on}, the inductor current i_{L1} build up .During t_{off} C_1 charges up by the current in L_1 and the dc source V_i ,while diode d is conducting the positive polarity voltage of the left side plate of C_1 rises, i_{L1} falls. During t_{on} C_1 discharges through L_2 , reverse biasing the diode d and charges the capacitor C_2 with its lower plate becoming positively charged. The inductor current i_{L2} rises during this time, as does i_{L1}. For large enough values of capacitors C_1 and C_2 during the switching period T_s leads to output voltage (v_{C2}) to be constant for a given switching time. This voltage changes with changing the switching period **(**t_{on}).



Fig(1) Circuit of Cuk Converter

3 – State Space Representation of the System

The differential equations which may be got, applying Kirichoffs voltage law to the circuit shown in Fig(1) for the two cases, , the switch is on or off, may be put into general state space form as follows:

$$x' = A\,x + B\,u \quad \text{.....(1a)}$$

$$y = C\,x + D\,u. \quad \text{.....(1b)}$$

where,
x = state vector (n-dimensional vector)
u = input vector (r-dimensional vector)
y = output vector (m-dimensional vector)

i_{L1} R_1 L_1 v_C

C_1

V_i S

A = $n\times n$ matrix
B = $n\times r$ matrix
C = $m\times n$ matrix
D = $m\times r$ matrix

The state variables for the Cuk converter circuit are assigned to be $x_1 = i_{L1}$, $x_2 = v_{C1}$, $x_3 = i_{L2}$, $x_4 = v_{C2}$.

From the loop equations of the circuit in Fig(1) , for the two states , switch is on and switch off, the following state and output equations may be derived: (Assuming ideal switch characteristics)

$$x_1' = -\frac{R_1}{L_1}x_1 - \frac{(1-\alpha)}{L_1}x_2 + \frac{V_i}{L_1} \quad \text{....(2.1)}$$

$$x_2' = \frac{(1-\alpha)}{C_1}x_1 - \frac{\alpha}{C_1}x_3 \quad \text{....(2.2)}$$

$$x_3' = \frac{\alpha}{L_2}x_2 - \frac{R_2}{L_2}x_3 - \frac{1}{L_2}x_4 \quad \text{....(2.3)}$$

$$x_4' = \frac{1}{C_2}x_3 - \frac{1}{R_L C_2}x_4 \quad \text{....(2.4)}$$

$$y = x_4 \quad \text{....(3)}$$

or

$$A = \begin{bmatrix} -\frac{R_1}{L_1} & -\frac{1-\alpha}{L_1} & 0 & 0 \\ \frac{1-\alpha}{C_1} & 0 & -\frac{\alpha}{C_1} & 0 \\ 0 & \frac{\alpha}{L_2} & -\frac{R_2}{L_2} & -\frac{1}{L_2} \\ 0 & 0 & \frac{1}{C_2} & -\frac{1}{R_L C_2} \end{bmatrix} \qquad B = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = [0\ 0\ 0\ 1] \quad , \quad D = [0]$$

Where α be a symbol of the duty cycle of the circuit

$$\alpha = \begin{cases} 0 & \text{switch } S \text{ is in position } 0 \\ 1 & \text{switch } S \text{ is in position } 1 \end{cases}$$

The block diagram of the system is shown in Fig(2).

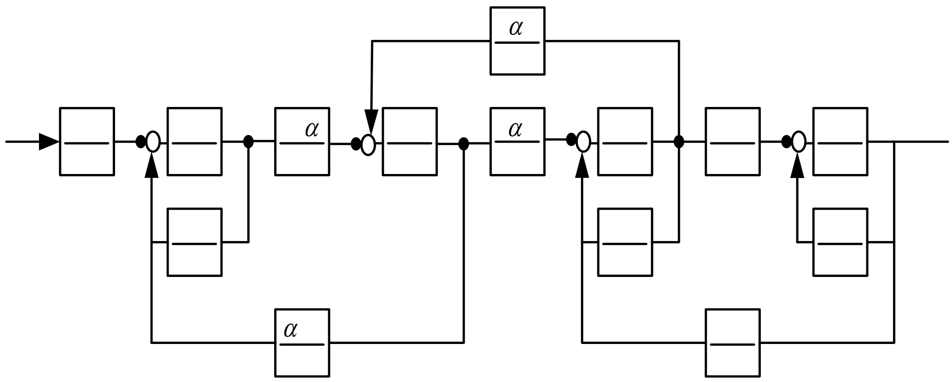


Fig. 2 : Block Diagram of System State Equations of the Cuk Converter

4 - System Uncertainty Description and Representation

The four parameters L_1,L_2,C_1, and C_2 in the circuit of Fig(1) , whose values are not approved precisely, but are theoretically assumed to be positioned in the following known interval: This interval is *25* % of their nominal values. The perturbations Δ_{L1}, Δ_{L2} , Δ_{C1} , Δ_{C2}, are introduced . They are assumed to lie between the interval [-1,1].

The sketch of (1+δΔ) can be attached to the selected parameters of the system in Fig(2) to represent the uncertainty variation of the system in the block diagram with uncertainty as in Fig(3)

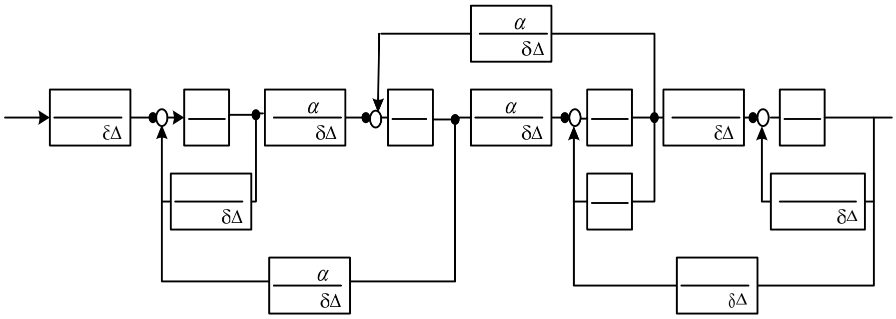
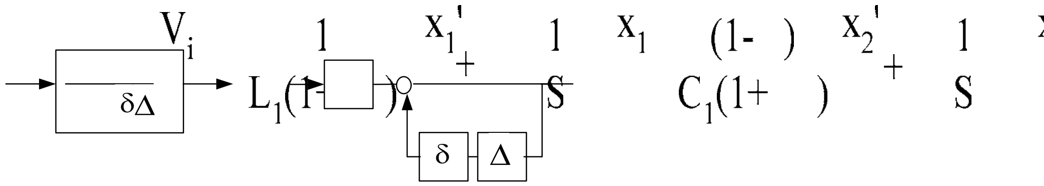


Fig. 3: Block Diagram of the System State Eq. of the Cuk Converter Circuit with Uncertainty

The block diagram of Fig (3) may be expanded so that each block containing perturbation to be presented as follows:



and the overall new block diagram will become as in Fig (4).

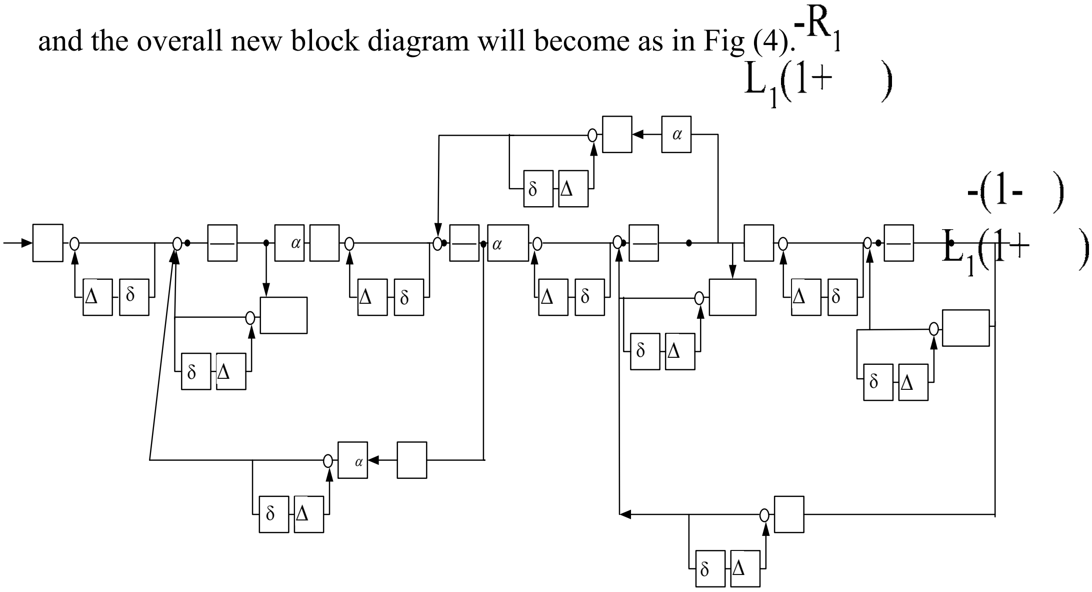


Fig. 4 : Block Diagram of Expanded System State Eq. of Cuk Converter Circuit with Uncertainty**.**

In order to apply the linear fractional transformation, the following steps must be followed:

1- defining the inputs and the outputs of the system:

The input of the dynamical system is V_i. The output is $y=v_{C2}$.

2- isolating the parametric uncertainties and, marking the inputs and outputs of $\Delta_{L1}, \Delta_{L2}, \Delta_{C1}, \Delta_{C2}$ as:

$[i_1,i_2,i_3],[i_4,i_5],[i_6,i_7,i_8],[i_9,i_{10}]$ and $[o_1,o_2,o_3],[o_4,o_5],[o_6,o_7,o_8],[o_9,o_{10}]$ respectively ,as shown in Fig(4).

Now with these parameters uncertainties, isolated, the whole system, has 11 inputs which are: $i_1,i_2, \ldots i_{10}$, V_i, and 11 outputs which are: $o_1,o_2,o_3 \ldots\ldots o_{10}$,y

The mathematical relations between all inputs and outputs in matrix form may be written as bellows:

Output = P matrix X input

or

$$
\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \\ o_1 \\ o_2 \\ o_3 \\ o_4 \\ o_5 \\ o_6 \\ o_7 \\ o_8 \\ o_9 \\ o_{10} \\ y \end{bmatrix}
=
\begin{bmatrix}
-\frac{R_1}{L_1} & -\frac{(1-\alpha)}{L_1} & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{L_1} \\
\frac{(1-\alpha)}{C_1} & 0 & -\frac{\alpha}{C_1} & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\alpha}{L_2} & -\frac{R_2}{L_2} & -\frac{1}{L_2} & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{C_2} & -\frac{1}{R_L C_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & -\delta_{L1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\delta_{L1}}{L_1} \\
\frac{R_1\delta_{L1}}{L_1} & 0 & 0 & 0 & 0 & -\delta_{L1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{(1-\alpha)\delta_{L1}}{L_1} & 0 & 0 & 0 & 0 & -\delta_{L1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{(1-\alpha)\delta_{C1}}{C_1} & 0 & 0 & 0 & 0 & 0 & 0 & -\delta_{C1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\alpha\delta_{C1}}{C_1} & 0 & 0 & 0 & 0 & 0 & -\delta_{C1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\alpha\delta_{L2}}{L_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta_{L2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{R_2\delta_{L2}}{L_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta_{L2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\delta_{L2}}{L_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta_{L2} & 0 & 0 & 0 \\
0 & 0 & \frac{\delta_{C2}}{C_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta_{C2} & 0 & 0 \\
0 & 0 & 0 & \frac{\delta_{C2}}{R_L C_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta_{C2} & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \\ i_9 \\ i_{10} \\ V_i \end{bmatrix}
$$

5 - Robust Performance of the Uncertain System

After representing the structured uncertainty of the parameters in the system. Some performance will be added to the system like input and output performance as shown in Fig(5).

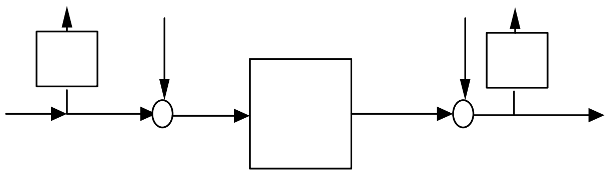


Fig. 5 : System Cuk Converter circuit with full perturbations

where,

W_{pert} weighting uncertainty for input circuit.

W_p weighting uncertainty for output circuit.

Applying LFT the general system matrix may be got, and two more rows and columns will be added to it. The following system matrices may be deduced from P (the general system matrix)

$$A=\begin{bmatrix} -\frac{R_1}{L_1} & -\frac{1-\alpha}{L_1} & 0 & 0 \\ \frac{1-\alpha}{C_1} & 0 & -\frac{\alpha}{C_1} & 0 \\ 0 & \frac{\alpha}{L_2} & -\frac{R_2}{L_2} & -\frac{1}{L_2} \\ 0 & 0 & \frac{1}{C_2} & -\frac{1}{R_LC_2} \end{bmatrix},$$

$$B_1=\begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$B_2=\begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad B_3=\begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

z

pertin

W_{pert}

V_i

+

+

+

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{R_1\delta_{L1}}{L_1} & 0 & 0 & 0 \\ 0 & \frac{(1-\alpha)\delta_{L1}}{L_1} & 0 & 0 \\ \frac{(1-\alpha)\delta_{C1}}{C_1} & 0 & 0 & 0 \\ 0 & 0 & \frac{\alpha\delta_{C1}}{C_1} & 0 \\ 0 & \frac{\alpha\delta_{L2}}{L_2} & 0 & 0 \\ 0 & 0 & \frac{R_2\delta_{L2}}{L_2} & 0 \\ 0 & 0 & 0 & \frac{\delta_{L2}}{L_2} \\ 0 & 0 & \frac{\delta_{c2}}{C_2} & 0 \\ 0 & 0 & 0 & \frac{\delta_{c2}}{R_L C_2} \end{bmatrix} \qquad C_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & W_p \end{bmatrix} \qquad C_3 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D_{11} = \begin{bmatrix} -\delta_{L1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\delta_{L1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\delta_{L1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\delta_{C1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\delta_{C1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\delta_{L2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\delta_{L2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta_{L2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta_{C2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta_{C2} \end{bmatrix} \qquad D_{12} = \begin{bmatrix} \frac{\delta_{L1}}{L_1} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$D_{13} = \begin{bmatrix} \frac{\delta_{L1}}{L_1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$D_{21} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{22} = \begin{bmatrix} 0 & 0 \\ 0 & W_p \end{bmatrix} \quad D_{23} = \begin{bmatrix} W_{pert} \\ 0 \end{bmatrix} \quad D_{31} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{32} = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad D_{33} = \begin{bmatrix} 0 \end{bmatrix}$$

6 - General Problem Description

All uncertain closed - loop systems may be represented as in Fig (6a,b). The following three basic components for the uncertain system are recognized:

1- general system interconnecting matrix P;
2- the controller K;
3- the uncertain elements and performance.

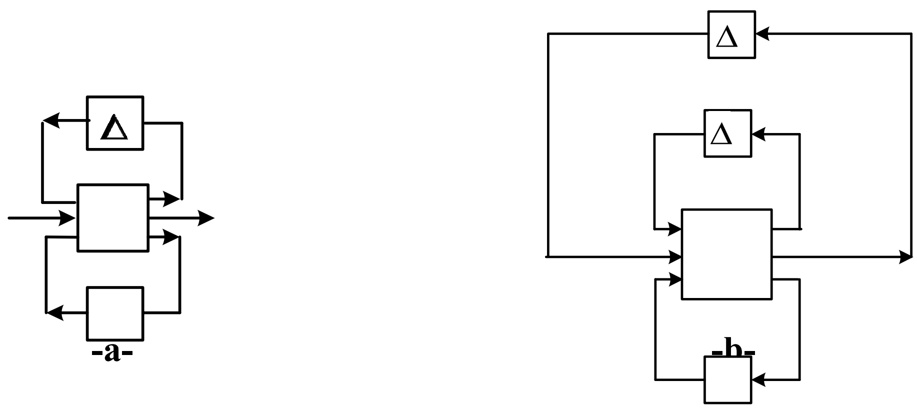


Fig 6 : The General Problem Description.

The recurrent fractions for the general system matrix P consists of recognizing pairs of input - output variables:
(u(t),y(t)) control and measurement variables.
(d(t),e(t)) disturbance and error signals.
(i(t),o(t)) the perturbation elements input, output signals which are attached in to the system throughout a norm bounded perturbation Δ(structure uncertainty).

The design object is to find a compensator K, which internally stabilizing the general system P, while keeping the matrix transfer function between d and e appropriately small for the whole set of allowable perturbation Δ

In the transformation process, from the classical setup into the more general setup, any uncertainty arising at systems components (actuator ,plant, sensors, ...

etc).level becomes automatically structured at the level of the generalized system P. this generalized system description as set in Fig(6a) is appropriate for the synthesis as well as for the analysis problem.
Definition: Given a generalized plant

$$P = \begin{bmatrix} A & B_1 & B_2 & B_3 \\ C_1 & D_{11} & D_{12} & D_{13} \\ C_2 & D_{21} & D_{22} & D_{23} \\ C_3 & D_{31} & D_{32} & D_{33} \end{bmatrix}$$

The matrix P to be partitioned consistently with Fig(6b), the following two standard assumptions may be made:

i) (A,B_3) is stabilizable and (C_3,A) is detectable.
ii) $D_{33} = 0$.

Assumption (i) is necessary and sufficient for the existence of an internally stabilizing output feedback controller. Assumption (ii) incurs no loss of generality but considerably simplified calculation [10].

Where,

$A = n\times n$ system matrix
$B_1 = n\times w$ structure input matrix, $B_2 = n\times d$ performance input matrix,
$B_3 = n\times r$ system input matrix, $C_1 = e\times n$ structure output matrix,
$C_2 = z\times n$ performance output matrix, $C_3 = m\times n$ system output matrix,
$D_{11} = e\times w$ matrix, $D_{12} = e\times d$ matrix, $D_{13} = e\times r$ matrix
$D_{21} = z\times w$ matrix, $D_{22} = z\times d$ matrix, $D_{23} = z\times r$ matrix
$D_{31} = m\times w$ matrix, $D_{32} = m\times d$ matrix, $D_{33} = m\times r$ matrix

7 - μ - Synthesis Theory

The robust performance problem can be worked out using μ- synthesis, which implies the minimization of the following criteria:

$$\min_K \inf_{D\in \underline{D}} \sup_{\omega} \overline{\sigma}\left(DF_l(P,K)D^{-1}\right)$$

Since this minimization problem can not be solved directly, an approximate method , usually referred to as D_s - K iteration is used. The criteria above are minimized for either K or Ds while holding the other constant. When D_s is hold constant ,the problem *is* solved by H_∞ optimization. For K is constant μ- analysis is carried out ,i.e. D_s is found such that the upper bound on μ is minimized. An invertible, stable minimum phase transfer function is fitted to the resulting frequency dependent D_s and augmented to the system description. This step increase the order of the controller

The iteration can be closed when the criteria (μ) is fewer than 1 or when its value stops decreasing.

7 – 1 D_s - K Iteration:

D_s - K iteration is applied to find the optimal robust performance of Cuk converter with uncertainties.

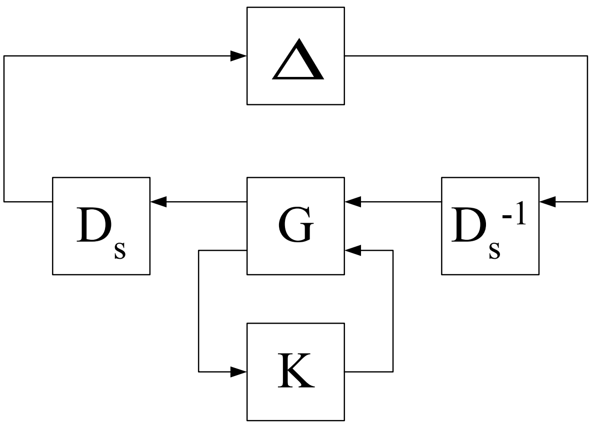


Fig. 7.1 : Structure of the D_s-K Iteration**.**

The procedure for μ - synthesis through D_s- K iteration involves several iterations. Each iteration consists of the following four steps:

1- H_∞- synthesis:

H_∞ - synthesis is applied on the generalized plant P matrix minimizing H_∞ - norm between (w, d) and (z, e),which means that the performance variables used in the H_∞ - synthesis are the combination of the original perturbation parameters and the original performance variables.

$$\hat{d} = \begin{pmatrix} w \\ d \end{pmatrix} \text{ and } \hat{e} = \begin{pmatrix} z \\ e \end{pmatrix}$$

If this step is performed for the first time, the D_s - scale is not yet in place and can be thought of to be an identity matrix making P the same as the original plant P.

2- μ - analysis**:**

Connecting the H_∞ -controller found in step (1) to the generalized plant yield $M = F_l\,(P,K)$. Choosing the structure of the total Δ- matrix, Δ_u as uncertainty block augmented with an extra performance block Δ_p .Performing a μ - analysis on M with structure Δ at a grid of frequency point. The results of the analysis are the upper and lower μ - bounds and the D_{sw} scaling matrix at every frequency point. If the peak value of μ does not exceed unity, the robust performance objectives are achieved and the μ - synthesis procedure can be stopped. Otherwise the μ - synthesis continues until the peak value of subsequent μ no longer decreases.

3 - D_s - scaling fitting**:**

The D_{sw} - scales found in the μ - analysis step consists of a constant complex matrix for every point of the specified frequency grid. When D_s are plotted against frequency these matrices can be fitted with a real rational, stable, minimum - phase transformation matrix $D_s(s)$.

If it is not the first time this step is performed, the previous fit has to be incorporated into D_s .The last block of a D-scaling matrix in a μ- calculation could be normalized to one. This means that if the

Δ - matrix consists of n - blocks, only n-1 need to be fitted with a transfer function.

4- addition of the D_s - scales to P_{i+1} :

Constructing the model for the P_{i+1} generalized plant by scaling the original P with the rational D_s - scale. The D_s - scale only scales the upper two pairs of variables of P_n. The total system matrix P can be scaled by augmenting the D_s - scale with an identity matrix at the (u,y) variables:

$$P_{i+1} = \begin{bmatrix} D_s & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} D_s^{-1} & 0 \\ 0 & I \end{bmatrix}$$

This terminates one iteration of a μ - synthesis procedure .The next step is to go back to step 1 and start the next iteration by performing a H_∞ - synthesis. The scaling of P with the rational D_s scales is used to trick the following H_∞ - synthesis to concentrate more on the upper bounds of μ than on the H_∞ - norm. In this way, after several iterations , the peak value of the upper bound of μ is minimized. Therefore the robust performance is equivalent to μ_Δ (M) or μ - synthesis constructs a controller K with the best possible robust performance. [11]

7 – 2 Commands for D_s – K iteration:

Within μ-tools there are four ways to perform D_s - K iteration. The one which is used here is through using the graphical user interface **dkitgui** for automated, adjustable, and visual iterations, which allows easy monitoring of progress .[11] shows how this is used to perform D_s - K iteration.

In order to construct the interconnecting matrix the interconnection program **sysic** from MATLAB package is used. Appendix C in [12].

Using the previous program the open - loop interconnection structure is entered with all entries and replacing the structure of the perturbation by: blk=[1 1;1 1; 11; 11; 11; 11; 11; 11; 1 1;1 1] and the performance block as [2 2]. Some bounds of δ are listed to see how one can get robust optimization for the Cuk converter circuit whose elements values are given in table 1. Let: δ_{L1}=19%, δ_{L2}=25% , δ_{C1}=25%,δ_{C2}=25% ,and δ_{pert} = 1 % , δ_p = 5%. Running the D_s - K iteration command in MATLAB package , the robust controller was obtained at the third iteration, where the value of μ becomes to be less than one . A 38 states controller was obtained exactly (μ_Δ (M) =O.994).

Table 2 shows the different maximum changes in the circuit parameters for the robust performance system.

Table 1 Parameters and Values of Cuk Converter Circuit

Parameters	Values(nominal)	Units	Description
L_1	3	mH	inductance
L_2	2	mH	inductance
C_1	200	μF	capacitance
C_2	100	μF	capacitance
V_i	20	V	Input voltage
R_1	0.2	Ω	Lose in L_1
R_2	0.2	Ω	Lose in L_2
R_L	5	Ω	load

Table 2 Peak μ Values for Different Numbers of Iterations.

No.	Perf.Unc.%		Structure Unc.%				No. of iteration	Controller order	Peak μ value
	W_{pert}	W_p	δ_{L1}	δ_{C1}	δ_{L2}	δ_{C2}			
1	5	1	25	25	25	25	5	36	1.0508
2	5	1	19	25	25	25	3	38	0.994
3	5	1	25	19	25	25	3	36	0.995
4	5	1	25	25	20	25	3	38	0.992
5	5	1	25	25	25	16	3	36	0.986
6	5	1	74	0	0	0	5	34	0.994
7	5	1	0	88	0	0	3	28	0.992
8	5	1	0	0	71	0	6	46	0.997
9	5	1	0	0	0	85	6	40	0.997

Fig (7.1-3) shows the progress in the values of μ during different iterations.

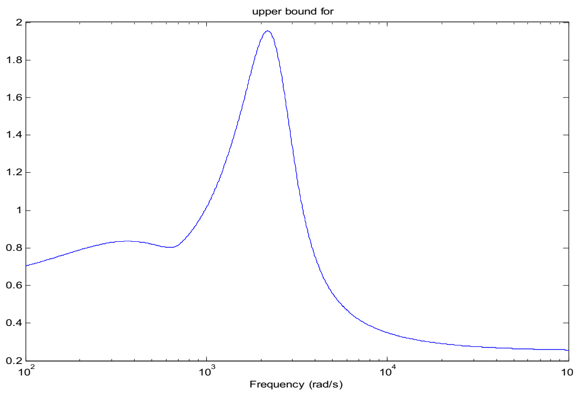


Fig. 7.1 : 1st-Iteration

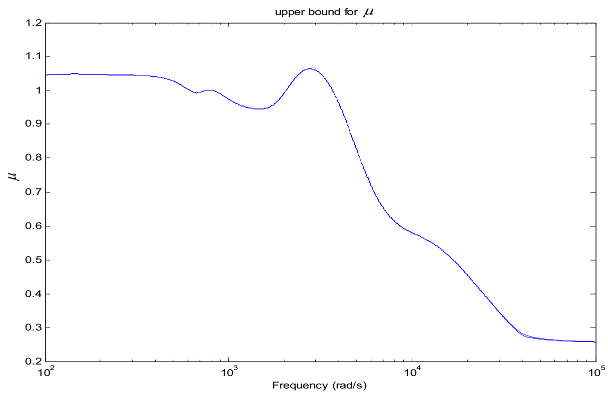


Fig. 7.2 : 2nd-Iteration

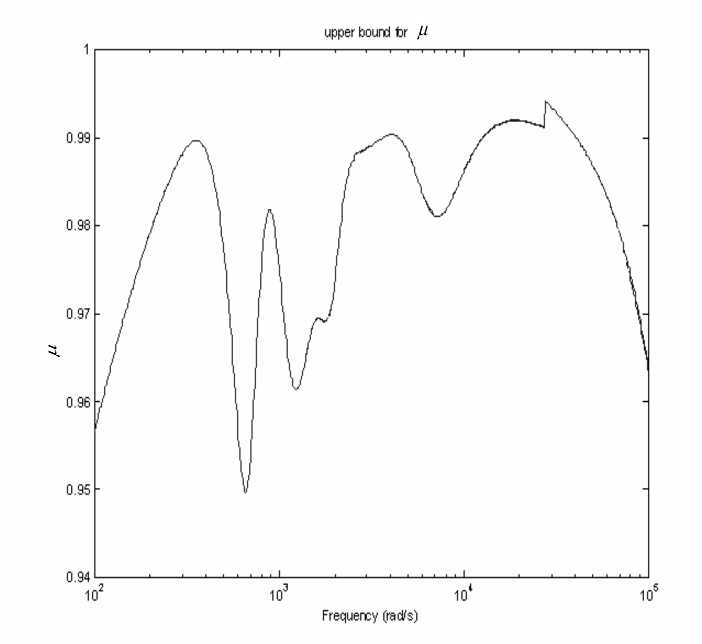


Fig. 7.3 : 3rd-Iteration

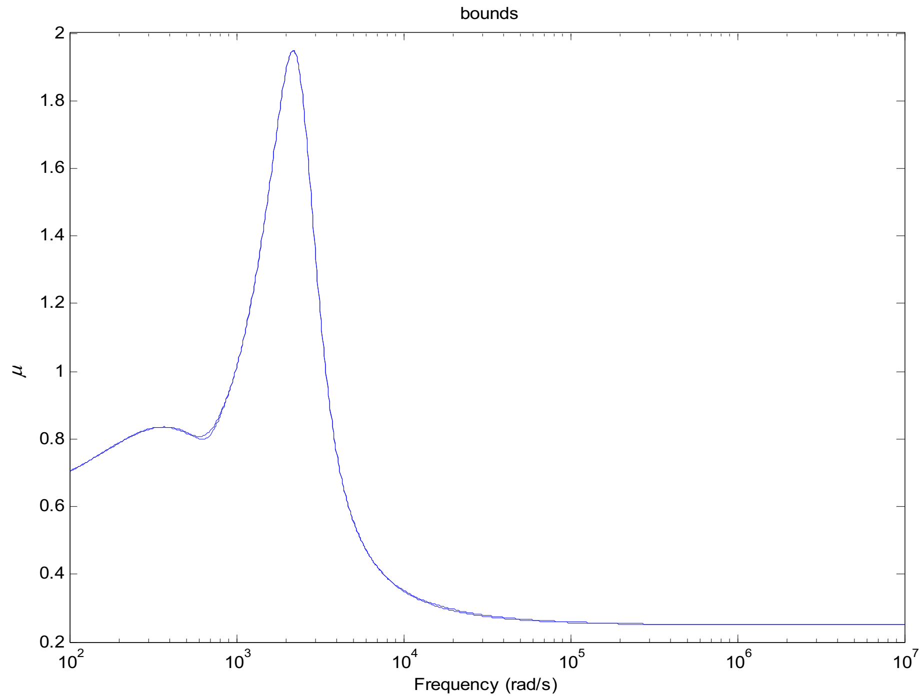


Fig. 8: Robust Performance μ.

8 - Results and Discussion

The results may be classified into two measure parts:

Pt. 1: The results obtained from applying H_∞ - design technique and μ - analysis may be summarized as follows:

The results of the closed - loop system with H_∞ controller design in Fig(8) show that the system is unstable for the given perturbations, with stable nominal performance. Fig(8) shows the effect of variation in frequencies on the system stability. The worst case is for frequency (1120 H_z) which give $\mu = 1.948$. The effect decreases, with increase of the frequency. The system is unstable, because H_∞ - design technique do not include the structured uncertainty

Pt.2: The results from applying μ - design technique to the system (Cuk converter) are:

From table 2 the following observations may be made:
-For the first set of perturbations, the peak value of μ for the fifth iteration is 1.0508. This means that at this uncertainty level the system is not guaranty the robustness and may become unstable.

- The maximum allowed variation in each parameter for robust stability is different from one element to another.
  - Numbers of iterations for each variation are different.
  - The order of controllers obtained, increases with increase the number of iterations.
  - Maximum allowed simultaneous perturbations for the four elements is 25%.

Fig (7.3) shows that the peak value of μ is less than one, which means that the system is robustly stable. The effect of frequency variation is reduced and can be neglected.

9 - Conclusions

Mathematical model in state space form is derived for DC– DC (Cuk) converter circuit.

The uncertainties are added to the system, inserting perturbation to the circuit parameters . Applying the description of LFT which , includes: formulating block diagram of Cuk converter circuit .Inserting the perturbations to its block diagram, and developing the general system matrix. This general system matrix includes description of all uncertainties and system different matrices (system, input, output matrices)

The controller for Cuk converter circuit with H_∞ - technique is designed, the closed - loop system with controller is seen to be robustly unstable, which means that $H\infty$ - design technique does not give robustness for structured uncertainty. To overcome the stability robustness problem, μ - design technique is applied to design a controller for the system. Analyzing the system with the new controller the following results where observed:

- The maximum allowed perturbation in the parameters which keep the system stable are as follows:

Δ_{Ll}=25 %, Δ_{C1}=25%, Δ_{L2} = 2O% and Δ_{C2}=25%.

- The individual maximum allowed perturbations for the above elements (L_1,C_1,L_2 and C_2) maintaining others constant are :δ_{Ll}=74%, δ_{C1} =8l%, δ_{L2}=7l% and δ_{C2}=85%.These perturbations values mean that the most sensitive element to perturbation is L_2.

The effect of the frequency variation on the stability of the closed – loop system may be neglected.

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